Markov Process

- A stochastic process is a function whose values are random variables
- The classification of a random process depends on different quantities
  - state space
  - index (time) parameter
  - statistical dependencies among the random variables *X(t)* for different values of the index parameter *t*.

Markov Process

- State Space
  - the set of possible values (states) that *X(t)* might take on.
  - if there are finite states => *discrete-state process* or *chain*
  - if there is a continuous interval => *continuous process*
- Index (Time) Parameter
  - if the times at which changes may take place are finite or countable, then we say we have a *discrete-(time) parameter* process.
  - if the changes may occur anywhere within a finite or infinite interval on the time axis, then we say we have a *continuous-parameter* process.

Markov Process

- In 1907 A.A. Markov published a paper in which he defined and investigated the properties of what are now known as Markov processes.
- A Markov process with a discrete state space is referred to as a *Markov Chain*
- A set of random variables forms a Markov chain if the probability that the next state is $S_{(n+1)}$ depends only on the current state $S_{(n)}$, and not on any previous states

Markov Process

- States must be
  - mutually exclusive
  - collectively exhaustive
- Let $P_i(t)$ = Probability of being in state S_i at time t.

$$\sum_{\forall i} P_i(t) = 1$$

- Markov Properties
  - future state prob. depends only on current state
    - independent of time in state
    - path to state

Markov Process

- Assume exponential failure law with failure rate λ.
- Probability that system failed at $t + \Delta t$, given that is was working at time t is given by

$$1 - e^{-\lambda \Delta t}$$

with

$$e^{-\lambda \Delta t} = 1 + (-\lambda \Delta t) + \frac{(-\lambda \Delta t)^2}{2!} + \cdots$$

we get

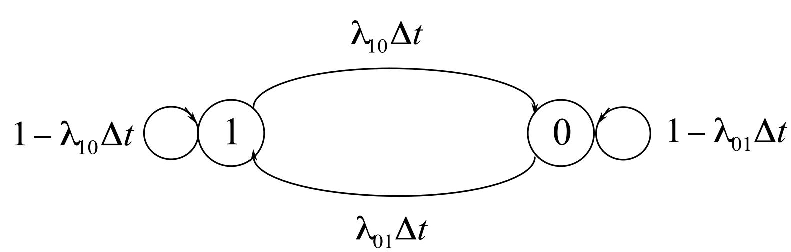
$$1 - e^{-\lambda \Delta t} = 1 - [1 + (-\lambda \Delta t) + \frac{(-\lambda \Delta t)^2}{2!} + \cdots]$$

$$= \lambda \Delta t - \frac{(-\lambda \Delta t)^2}{2!} - \cdots$$

Markov Process

- For small Δt

$$1 - e^{-\lambda \Delta t} \approx \lambda \Delta t$$



Markov Process

- Let P(transition out of state i in Δt) =

$$\sum_{j \neq i} \lambda_{ij} \Delta t$$

- Mean time to transition (exponential holding times)

$$\frac{1}{\sum_{j \neq i} \lambda_{ij}}$$

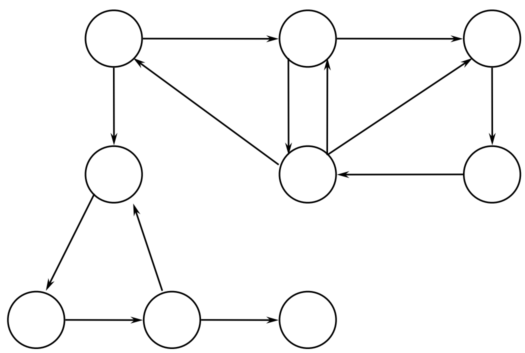
- If λ's are not functions of time, i.e. if $\lambda_i \neq f(t)$
  - homogeneous Markov Chain

Markov Process

- Accessibility
  - state S_i is accessible from state S_j if there is a sequence of transitions from S_j to S_i.
- Recurrent State
  - state S_i is called recurrent, if S_i can be returned to from any state which is accessible from S_i in one step, i.e. from all immediate neighbor states.
- Non Recurrent
  - if there exists at least one neighbor with no return path.

Markov Process

- sample chain



Which states are recurrent or non-recurrent?

Markov Process

- Classes of States
  - set of states (recurrent) s.t. any state in the class is reachable from any other state in the class.
  - note: 2 classes must be disjoint, since a common state would imply that states from both classes are accessible to each other.
- Absorbing State
  - a state S_i is absorbing iff

$$\sum_{j \neq i} \lambda_{ij} \Delta t = 0$$

Markov Process

- Irreducible Markov Chain
  - a Markov chain is called irreducible, if the entire system is one class
    - => there is no absorbing state
    - => there is no absorbing subgraph, i.e. there is no absorbing subset of states